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Bach's extraordinary temperament: our Rosetta Stone—2

As Mark Lindley remarked about the study of temperament systems:

> The only real test is the sound. On several occasions when I have tuned a suitable instrument in an historically likely manner and then tried out some part of the appropriate repertory for the first time, I have met with surprises; and always I have heard something, some effect in the harmony, some rapport between a nuance of the tuning and the instrument's timbre, which could not be anticipated from looking at the score. . . . As we look for a convincing pattern of distribution among the relatively pure and impure chords (and among the relatively dull and incisive semitones), we are led again to tune an appropriate instrument and discover the musical effect. A certain kind of scholar will complain—indeed has complained—that this method is too subjective, even when accompanied by the other kinds of evidence described here. I think it worthwhile, however, to use our ears as best we can, and hope for a well-informed consensus to confirm our perceptions, or improve upon them.[1]

The best test *is* indeed the sound. The second part of this article is much enhanced if a harpsichord is available for direct listening to the intervals, as analysed here. There really is no substitute for hands-on experience with this, trying all the intervals for oneself.[2]

Melodic and harmonic characteristics: overview

To recapitulate from part 1: Bach's keyboard bearing has five ⅙ comma 5ths F–C–G–D–A–E, then three pure 5ths E–B–F♯–C♯, and finally three 1/12 comma 5ths C♯–G♯–D♯–A♯. This results in a slightly wide (1/12 comma) A♯–F.

Listening to keyboards set in Bach's tuning, it becomes obvious: in his usage, the word *wohltemperirt* meant this specific practical alteration of normal temperament (i.e. regular ⅙ comma). His explicit adjustments—the uncommonly high placement of pure and half-tempered 5ths—make all the keys pleasing and usable in musical practice: yielding a 'well-tuned keyboard' on which everything works fine. This was a main point of *WTC*: the example of one prelude and fugue in every possible key, as J. K. F. Fischer had also done (with most keys) in *Ariadne musica* (1700 and 1715).[3] As C. P. E. Bach and J. F. Agricola put it in Bach's obituary: 'In the tuning of harpsichords, he achieved so correct and pure a temperament that all the tonalities sounded pure and agreeable. He knew of no tonalities that, because of impure intonation, one must avoid.'[4]

The tonal relationships are exciting: C major and F major remain the best in tune, E major is the most brilliant key, and there is no harshness anywhere. The B–D♯ and F♯–A♯ major 3rds, so prominent in dominant triads in Bach's music, are sweeter and lighter than E–G♯. The D♭–F and A♭–C 3rds, also used frequently,[5] are much more mellow than their counterparts in most other temperaments.[6] As we shall see later, melodies have no obtrusively large or small steps, but subtly expressive inflections.

Every major and minor key is immediately distinct: in the intonation quality of its intervals, both melodically and in chords. Through close listening, experience and study one may perceive the *objective* character of each tonality.[7] This latter property is familiar from some other 17th- and 18th-century circulating temperaments, but Bach's *asymmetric* organizing pattern (peaking at E major) is exceedingly rare.[8] It is this balanced irregular shape that gives tonal music such agreeable flexibility through all 24 major and minor keys: all keys are equally usable and therefore equally pure (*gleich rein*, as C. P. E. Bach put it).

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Playability of Bach's keyboard repertory

Notoriously problematic Bach pieces for keyboard players/tuners include: both books of *WTC*; the B minor Partita (BWV831, originally in C minor); the E♭ (BWV819) and F minor (BWV823) Suites; the C minor 'French' Suite (BWV813); the Toccatas (BWV910–16, especially 910–12); the six violin sonatas (BWV1014–19); the B♭ (BWV992) and E major *Capriccii* (BWV993);[9] all of C*lavierübung III* (see the website); the Chromatic Fantasy and Fugue (BWV903); other *Fantasien* (BWV 562, 904, 906, 917, 918, 919, 922, 944); the Sarabande of the G minor 'English' Suite (BWV808); the C minor Partita for lute or keyboard (BWV997); the harpsichord concertos in E major (BWV1053 and cantatas 49 & 169) and F minor (BWV1056 and cantata 156); *O Mensch, bewein dein' Sünde gross* (BWV622; see the website); *Kleines harmonisches Labyrinth* (BWV591; see the website); the G minor Fantasia and Fugue (BWV542);[10] the Praeludium/Toccata in either E or C (BWV566);[11] the Praeludium and Fugue in C (BWV547); the C minor fugue (BWV575); the C minor section of the F major Pastorella (BWV590); 'Goldberg' variation 25 of C*lavierübung IV*; the two *ricercare* and the *canon per tonos* of *Das musikalisches Opfer*.[12]

Remarkably, these are all free of problems in Bach's temperament. Have some of these pieces been avoided in players' repertories *because of* tuning difficulties? I know that is true for me, and suspect that other players have also found the tuning puzzle a convenient impetus not to spend much time with these pieces. These compositions make us sound like incompetent keyboard tuners, and make our instruments sound sour, reflecting badly on us as musicians. But it is not Bach's fault that we had lost the correct manner to tune for them.

Beyond merely an avoidance of startling anomalies, these pieces and others reveal subtle effects whenever the music modulates.[13] The F minor Sinfonia (BWV795) and E minor Partita (BWV830) are exquisite. The second Bourrée of the A major English Suite (BWV806) is a startling contrast, as the bright sharps of the suite all suddenly exit. The Fantasie BWV922[14] simply must be experienced at the keyboard—no words can describe its adventures, in Bach's tuning.

From the beginning of Bach's career the toccatas in F♯ minor (BWV910),[15] C minor (BWV911) and D major (BWV912) also all shine brilliantly. This temperament solves a long-standing problem of playing all the toccatas satisfactorily in an unequal temperament, without retuning the instrument between them.[16] As Robert Marshall has argued,[17] all seven of the extant *manualiter* toccatas BWV910–16 are primarily organ pieces. This harmonically daring music implies that Bach already had his temperament, or something very similar, on at least some organs at his disposal when he wrote these pieces in 1707–13. At least half a dozen of the contemporary 'Neumeister' chorales[18] corroborate this: the harmonic progressions into extreme keys sound rough in Werckmeister's temperaments but fine in Bach's.[19]

Turning to music from Bach's last dozen years, *Die Kunst der Fuge* gives an interesting effect in this temperament: against the neutral background of D minor, all the accidentals stand out as dynamic flashes of colour, each different from the others.[20] Play-throughs of *Clavierübung III* and *Das musikalisches Opfer*[21] suggest that these books in a relative major/minor relationship[22] are packed (respectively) with Bach's theological and political commentaries[23] about their subjects: a glorious triune God, and the local earthly king (a military aggressor who preferred French to German culture, and who was notably anti-Christian).[24] The *Schübler* and Leipzig chorales[25] offer remarkable contrasts of *Affekt* within each set. The C major prelude and fugue BWV547, a Leipzig piece,[26] makes such prominent use of the notes A♭ and D♭ that competing unequal temperaments are doomed to crash. (And don't miss bar 57 of the fugue!)

Judged from the substantial evidence of his music, did Bach learn, discover or develop his specific tuning method by his early 20s, and continue using it for the rest of his life? I believe it is likely, from the perspective of playability in all his music, and from the inexhaustibly expressive resources of the temperament itself. I understand the dangers of speculating that this temperament might reach far on either side of 1722. But, on hearing how well it works in practice, I see no reason why Bach would ever have discarded such an effective solution for his musical purposes. It allows his music to sound beautiful, richly layered, and continuously engaging through balanced contrasts.

With this temperament on my stringed keyboards[27] I have played through all of *WTC* and most of Bach's other keyboard and organ music. I have also been exploring Pachelbel, J. K. F. Fischer, Froberger, Frescobaldi, Böhm and other earlier composers whose work Bach knew; much of François Couperin (especially the harmonically daring *ordres* 25–7 from 1730),[28] and C. P. E. Bach's sonatas and Wilhelm Friedemann Bach's polonaises. The contrasts of key quality are obvious whenever the music modulates around or skips across the circle of 5ths. After almost a year of this, I remain astonished at each new piece I play, in the way the temperament reveals things that are not obvious from the page. The music sounds new to me, even after 20 years of experience playing it in other mean-tone variants and circulating temperaments. This tuning has changed the way I phrase and articulate the music. Aesthetic considerations are not offered here as proof, but merely as an assertion that the musical results go beyond merely 'plausible' to 'exquisite'. Readers are urged to do the same with their instruments, and draw their own musical conclusions.

Three different 'equal' temperaments joined into one

Bach's temperament is an extraordinary blend of notes from three different sets, like carefully blended spirits. The 12 available notes are organized in three subsets, according to the interval relationships in each subset:

- regular ⅙ PC (55-note equal temperament): C, D, E, F, G, A [the current Italianate/French taste]
- Pythagorean (just intonation): B, C♯, E, F♯ [perhaps reflecting Bach's interests in antiquarian styles]
- 12-note equal temperament: E♭, G, A♭, B♭, D♭ [the moderation of good German uprightness to bind all styles into a unity]

These subsets have 6, 4 and 5 notes respectively, and with the overlap[29] all 12 notes are supplied. We have here three different systems of 'equal' temperament[30] intermixed in 6:5:4 proportion: the same proportion that represents the frequencies of a pure major triad.[31]

Bach's temperament is therefore a unification of traditional mean-tone principles (the focus on the quality of major 3rds, splitting the SC), equal-temperament principles (focusing on the 5ths, splitting the PC as smoothly as possible), and the ancient Pythagorean tuning of pure 5ths.

Regular ⅙ comma (dividing either comma), by itself, lacks the resource of free modulation, and its leading notes (diatonic semitones) are wide enough that the melodic motion of resolution is impaired.[32] Diatonic and chromatic semitones[33] are so different from one another (a truism of all the mean-tone and modified mean-tone varieties) that melodies using them can sound bumpy. Furthermore, it is static: all the usable triads sound the same as one another—provided that they are spelled correctly. It therefore favours repose over harmonic and melodic motion. The '55-division' nomenclature of the 18th century was simply a theoretical attempt to explain the common practice, rounding the messy business of commas to easily understandable integers (see below, in the discussion about enharmonics).

12-note equal temperament lacks key character. In effect, when tonal music in any major key is played in equal temperament, it never leaves the average key character of Bach's D or E♭ major: modulation within a composition to a new key accomplishes little beyond a change of pitch level.[34] C*lavierübung III* is a good example. When played in Bach's temperament, the prelude and fugue that 'bookend' the collection begin and end in equal temperament, but leave it whenever the music modulates. If the book is played in equal temperament throughout, there is nowhere to go on either side of that average sound, and an interesting dynamic dimension is removed from the music.

Pythagorean tuning gives diatonic semitones that are smaller than chromatic semitones, and therefore strong melodic tendencies: but it lacks consonant major 3rds. Whenever four consecutive 5ths are tuned pure within any temperament (for example, the flat side of the circle in Werckmeister's, Vallotti's, Kellner's, or Barnes's), the resulting major 3rd is harsh: a SC too wide.

An irregular or mixed temperament solves these problems: by giving enough melodic regularity, a recognizable variety to harmonies (both simple and complex) to keep the ear engaged, and the ability to use everything without encountering serious problems of intonation. Each interval is slightly out of tune by a tasteful and carefully organized amount,

so that none is terrible: the difficult goal of any temperament.

As Bach's solution puts all three of these competing 'equal' schemes into action simultaneously, in a delicate 6:5:4 balance, their strengths outweigh one another's liabilities: giving an extraordinary subtlety and interesting character for all tonal music, both harmonically and melodically. The tonalities are distinct—not interchangeable willy-nilly!—but they are also all equally usable. A composer or arranger may transpose all or part of a piece to a different key, thereby changing its 'personality' for another occasion or practical set of circumstances; and Bach (the eminently practical musician) frequently did so.

'Equal' tuning schemes (and the exact division of intervals into two equal portions, geometrically) go back to the demonstration by the 15th-century theorist Jacobus Faber Stapulensis.[35] I would not be surprised if the 'FABER/BACH' canon (BWV1078), with its suspicious-looking date of 1 March (New Year's Day) 1749, were *much* earlier,[36] and related to Bach's mid-life concerns about tuning issues. As we will see below, the tuning quality of the '*mi fa*' and '*fa mi*' semitones is paramount in the recognizable signature of each scale, and 'all of music' is generated by the careful handling of semitones and enharmonics.

Enharmonics

As Francesco Tosi pointed out in his 1723 manual of vocal instruction, singers must be able to recognize and perform the enharmonic difference of a comma in melodic contexts. A note such as E♭ is a comma higher than its partner D♯, and if the wrong one is performed it offends the ear. Furthermore, appoggiaturas and other passing notes must always use the correct diatonic (not chromatic) notes of the current harmony.[37] Each time we pass through 12 consecutive 5ths, we arrive a comma higher or lower than our previous position. That is, the basic standard of intonation is a spiralling scale of regular ⅙ comma 5ths, also known as the 55-division of the octave, where commas are the individual tiny steps of this scale. The diatonic semitone and chromatic semitone are in 5:4 ratio within a tone whose size is 9 commas. Therefore, of the 55 equal parts of the octave, the notes are placed on the keyboard as follows: C = 0, C♯ = 4, D = 9, E♭ = 14, E = 18, F = 23, F♯ = 27, G = 32, G♯ = 36, A = 41, B♭ = 46, B = 50, C = 55.[38] Other important notes missing from a regular ⅙ comma keyboard include D♭ = 5, D♯ = 13, E♯ = 22, G♭ = 28, A♭ = 37, A♯ = 45, C♭ = 51, B♯ =54. Sauveur, Quantz, Telemann, Leopold Mozart and other 18th-century musicians confirm this same standard in both practice and pedagogy.[39] A close reading of Quantz's chapter 'On the Duties of Those Who Accompany a Concertante Part'[40] suggests that *only* the accidentals are exceptionally tempered on keyboards (i.e. outside the equal comma points of the 55-division), while the naturals are regular.[41]

What does this tell us? If possible, let us put ourselves for the moment inside the heads of exceptionally good 18th-century performers, such as Tartini or Tosi or Quantz, and bring our instruments and voices to a harpsichord in some temperament. As we try to sing or play with it, does it sound in tune or out of tune with our expectations and our normal handling of commas? Does the harpsichord have any notes that make a jarring effect by being a comma or more too high or low, according to our standard of spiralling ⅙ comma 5ths across the entire 25-note range from E♭♭–B♭♭–F♭–C♭–etc.–C–G–etc.–F♯–C♯–etc.–B♯–F𝄪–C𝄪? Or are the compromises easy to find, moderate, and comfortable for one's own performance (whether one tries to match them exactly or not)?

Such a thing is easily measurable, and a useful way to analyse a temperament's suitability for tonal music, but to my knowledge it has not been published before. In the tuning literature, the measurements we usually see are deviations in cents from *equal temperament*,[42] which is irrelevant by the 18th-century non-keyboard standard.

The notes on a standard 12-note keyboard must serve double duty, well or badly as the case may be, in all the enharmonic contexts required of them: this is a basic expectation of circulating temperament. Here are comparative examples of various temperaments, next to Bach's. All the notes are measured from their positions in the 55-division system, i.e. a double cycle of regular ⅙ PC (or 2/11 SC), with the common note C always at 0 per cent.

See table 1. The main things to notice here are the comma errors of the less common accidentals,

Table 1 Comparison of enharmonic treatment

Bach 1722 ⅙ and 1/12 PC												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	66.7	75.0	100.0	100.0	100.0	100.0	−100.0	−100.0	−83.3	−66.7	−50.0	−41.7
Error % PC	−33.3	−25.0	0.0	0.0	0.0	0.0	0.0	0.0	16.7	33.3	50.0	58.3
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**
Equal temperament												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	75.0	83.3	91.7	100.0	108.3	116.7	−75.0	−66.7	−58.3	−50.0	−41.7	−33.3
Error % PC	−25.0	−16.7	−8.3	0.0	8.3	16.7	25.0	33.3	41.7	50.0	58.3	66.7
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**
Werckmeister III 1691 ¼ PC												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	50.0	66.7	83.3	100.0	91.7	83.3	−125.0	−108.3	−91.7	−100.0	−83.3	−66.7
Error % PC	−50.0	−33.3	−16.7	0.0	−8.3	−16.7	−25.0	−8.3	8.3	0.0	16.7	33.3
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**
Kellner 1975 ⅓ PC												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	50.0	66.7	83.3	100.0	96.7	93.3	−110.0	−113.3	−96.7	−100.0	−83.3	−66.7
Error % PC	−50.0	−33.3	−16.7	0.0	−3.3	−6.7	−10.0	−13.3	3.3	0.0	16.7	33.3
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**
'Vallotti' ⅙ PC												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	66.7	83.3	100.0	100.0	100.0	100.0	−100.0	−100.0	−100.0	−83.3	−66.7	−50.0
Error % PC	−33.3	−16.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	16.7	33.3	50.0
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**
Regular ⅙ PC												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	100.0	100.0	100.0	100.0	100.0	100.0	−100.0	−100.0	−100.0	−100.0	−100.0	−100.0
Error % PC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**
Neidhardt 1724 'Small city', 1732 'Big city' ⅙ and 1/12 PC												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	66.7	83.3	91.7	100.0	100.0	100.0	−100.0	−91.7	−75.0	−66.7	−58.3	−50.0
Error % PC	−33.3	−16.7	−8.3	0.0	0.0	0.0	0.0	8.3	25.0	33.3	41.7	50.0
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**
Sorge 1758 ⅙ and 1/12 PC												
Enharmonic	D♯	A♯	E♯	B♯	F𝄪	C𝄪	B𝄫	F♭	C♭	G♭	D♭	A♭
Enharm. error % PC	66.7	75.0	83.3	100.0	100.0	100.0	−100.0	−91.7	−75.0	−66.7	−58.3	−41.7
Error % PC	−33.3	−25.0	−16.7	0.0	0.0	0.0	0.0	8.3	25.0	33.3	41.7	58.3
Primary function	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**

i.e. the top row of percentages in each. A positive percentage shows the amount by which that note is too sharp, and a negative percentage shows flatness. Some observations from the chart:

In regular ⅙ comma, the 12 main notes from E♭ to G♯ are all exactly in tune (0 per cent comma deviation) and the other 12 are 100 per cent comma either too high or low. This confirms Tosi's pedagogical analysis of the situation: for example, where the music says 'D♭', if we took our pitch from the keyboard, we are singing it 100 per cent of a comma too low, because the keyboard has it tuned primarily as a C♯.

In equal temperament, enharmonics are organized symmetrically in pairs outward from C in

both directions. Because the progression around the 5ths is smooth and steady, no single notes ever stick out as 'wrong'. The sharps are increasingly bright (and uncommonly high in general) while the flats are mellow (being only a few positions around the spiral of 5ths from C). D♭ and A♭ are quite well in tune, while their counterparts C♯ and G♯ are much higher in their own contexts; this creates an effect of brilliance in sharp-key music.

Bach's temperament progresses outward from C, with the sharps rising more quickly than the corresponding lowering of the flats. This is its most unusual feature, and the reason why it works so uncommonly well for tonal music. Its crossover point of 50 per cent is at C♯/D♭; i.e. exactly half way, as major 3rds between A below and F above. D♭ therefore sounds suave (not quite wide enough from F to be harsh), while C♯ is exciting (very noticeably sharp, e.g. in D minor context). This temperament favours A♭ ahead of G♯; again emphasizing the smoothness of the flat and the brilliant colour of the sharp. Six of the seven naturals are exactly in tune, making it easy for our singer or instrumentalist to find them; the only exceptional attention must be paid to the treatment of accidentals, being some portion of a comma too high or low, in a smoothly organized pattern outward.

'Vallotti' also has a smooth and symmetric progression in both directions, and it obviously favours the quality of the 12 main notes from E♭ to G♯. G♯/A♭ is the crossover point of 50 per cent; D♭ and G♭ quickly become quite harsh in tonal contexts, as is familiar to keyboardists who play in this temperament frequently. (Music in C minor, F minor and A♭ major brings out some of the melodic flatness of those notes.) All seven naturals are exactly in tune by the 55-division standard, and the accidentals rise or fall gradually.

Sorge 1758 has a pattern similar to Bach's, but starts the rise and fall closer to C in each direction. C♯ is slightly lower (more settled as a major 3rd above A), at the expense of quality in the D♭. Again compared with Bach's, this temperament has the trade-off of slightly faster-beating major 3rds C–E, F–A and G–B, for the benefit of more harmonious E–G♯, A–C♯ and B–D♯, while preserving the same overall shape. In practice it sounds like the musical meeting of unequal-temperament expert Bach with equal-temperament expert Sorge (two consecutive members of the Mizler society in 1747), coming to the complex problem of tonal music from opposite sides and striking a brilliantly tuned compromise.

'Werckmeister III' and 'Kellner' share a serious problem in their treatment of melody, *vis-à-vis* the 55-division. This phenomenon, precisely, is the reason why these temperaments can seem so arbitrarily and suddenly sour when normal tonal music is played in them. Specifically, the notes D♭, G♭ and C♭ are placed much too low, and D♯ and A♯ too high. Harshness shows up in the Pythagorean major 3rds (actually the misspelled diminished 4ths) C♯–F, F♯–B♭, B–E♭ and G♯–C, the legacy of regular mean-tone layouts; but it also appears in melodic contexts having nothing to do with major 3rds. Melodic leaps up or down to A♭ and D♭ can come across like singing with poor breath support as the tuning of these notes is so unexpectedly low. But, so is the note A in the simple melody F–A–C![43] The misspelled notes simply stick out obtrusively, because their deviations from the 55-division skip across the set of notes rather than being well-organized outward from C. In Werckmeister III, the C and F♯(!) are best in tune; then the G, E and B. Then, the F, D and C♯; and so forth, leaping in ways that are not intuitively obvious. How are our singer and instrumentalist to find their pitches accurately, when the deviation patterns on the keyboard are clustered around *both* C and F♯?

These latter two temperaments have plenty of faithful and enthusiastic fans, especially due to the way they sound reasonably good in *earlier* music (based mostly on regular mean-tone layouts). But it cannot be denied that their melodic bumpiness borders on the effect of randomness, by the 18th-century standard itself. I suspect that musical ears more readily tolerate notes that are slightly sharp than notes that are noticeably flat within melodies.[44] I cannot explain that phenomenon adequately. I can, however, offer a suggestion as to the technical *cause* of this problem in Werckmeister III.

In a well-known remark in his *New Grove* 'Well-tempered clavier' article, Lindley has observed that Werckmeister III was probably designed for the conversion of existing organs from regular

quarter-comma mean-tone. Some of the pipes of each octave can be left at or near their original pitch, and the conversion will take less work and expense. Direct examination of frequency charts, always keeping C constant, shows that Werckmeister's scheme (by this hypothesis) is even more clever than that; it allows conversion from *any* of the regular ¼, ⅕ and ⅙ layouts by keeping C along with three or four other notes. These notes need to be moved a smaller distance than 2.5 cents (i.e. only an easy revoicing):

⅙ syntonic: leave C, C♯, F♯, G, B

⅙ Pythagorean: leave C, E, F♯, G, B

⅕ syntonic: leave C, E, F♯, G

⅕ Pythagorean: leave C, D, E, G

¼ syntonic: leave C, D, G, A

Viewed from this angle, the axis of C–F♯ also makes additional sense. F♯ is left at or near its position from regular ⅙ comma temperament, namely the tritone 45⁄32 above C, one syntonic comma flat! The temperament begins with C–F–B♭–E♭–A♭–D♭ – G♭ pure 5ths, establishing 'F♯' at one Pythagorean comma below C. Therefore the six remaining 5ths from C up to F♯ must absorb 100 per cent of this comma amongst themselves, since we have not done any tempering yet. The assignment of quarter-PC tempering to C–G–D–A and to B–F♯ is simply the pattern that allows the most purity to be preserved in C, G, D and F majors without making the major third C–E flatter than pure, and it is only *coincidental* that this spacing is a quarter of a [Pythagorean] comma. It is not really a quarter-comma temperament, in most of its organization.

It appears to me (revising Lindley's hypothesis) that the main purpose of this temperament is to convert regular ⅙ or ⅕ comma organs, not quarter-comma organs. Werckmeister himself had already remarked about the circulating temperament shape we know as 'Vallotti' (⅙ comma tempered 5ths F–C–G–D–A–E–B), referring to it as an ordinary Venetian temperament, in 1681: long before Vallotti, Tartini or Barca did.[45] Furthermore, Werckmeister IV is even more obviously a conversion temperament, squaring off regular ⅙ Pythagorean comma and splitting its wolf in two; and Werckmeister V is a conversion of regular ⅛ Pythagorean comma. In this perspective, I believe it is plausible that the familiar Werckmeister III started from the *premise* that C and F♯ should be left alone, and everything else arranged around them. The resulting shape of it, as I have demonstrated here, suggests as much: the F♯ and its nearest neighbours are serious liabilities in music that treats them enharmonically.

Kellner's temperament, in turn, was his attempt to take the model of Werckmeister III to the next steps that seemed logical to him, slightly moderating its intensity while keeping essentially the same pattern.[46] Devie, Rasch and Lindley have offered perceptive further comments about Kellner's methods, which I need not repeat here.[47] The resulting temperament has some attractive symmetries and balances for music that never strays far from the basic set of mean-tone notes, but it does not handle enharmonic equivalences gracefully; and therefore it sounds remarkably rough in Bach's music.

Buxtehude's extant organ music uses G♯, D♯, A♯, E♯ and B♯ with impunity, with occasional forays to F𝄪 and C𝄪. On the flat side it goes only as far as A♭ (which in context must be decent as a 5th against E♭) frequently, and there are only several pieces[48] that use D♭. That suggests to me that if Buxtehude's organ temperament(s) were regular, only something as light as ⅙ comma makes musical sense;[49] and if irregular, having at least a compromised G♯/A♭ and perhaps also a lowered E♭ and B♭.[50] Werckmeister III fits very well for music that avoids the flats as thoroughly as Buxtehude's does. Saying the same thing in another way: Buxtehude's *œuvre* (as corroboration) makes Werckmeister III look like a practical method to convert regular ⅙ comma 'mean-tone' to smoother chromaticism, but *only around the sharp side*.

To summarize my remarks about enharmonics: any keyboard temperament as a candidate to play Bach's music must be able to handle all 25 notes from E♭♭ up to C𝄪, gracefully and in a sequence that singers and instrumentalists can find without undue trouble. The same feature (and restriction) that makes a temperament good for accompaniment, namely its conformance to the 55-division, also makes it good for solo repertory, as the accidentals are constrained to be in moderate and logical positions.

This is a prerequisite to vocal-sounding melodic contours: never having any individual notes that protrude too noticeably from their melodic and harmonic contexts, in steps and leaps. If the notes within scales are not absolutely regular, the irregularities must be tastefully subtle lest they begin to sound like mere errors from incompetence. This is a difficult balance to achieve. The 12 available pitches must be viewed (heard) from all possible angles, and deliver something reasonable to every possible context.

Recognizable scales through the distinct intonation of their steps

As Ledbetter's book reminds us repeatedly, every key has a distinctive 'grip' (*Griff*) in the player's hands. The player must learn to grasp them all, and the *WTC* provides terrific examples. As Bach's temperament makes clear, that distinctiveness in physical layout on the keyboard also has a counterpart in distinctive intonation patterns in every key.[51] The contrasts, to the player, are not only psychological (from the physical *Griff*) but also audible. To the close listener (which is also a crucial component of *playing*), these differences in tension further affect phrasing and timing: perception of the motion in tonal music.

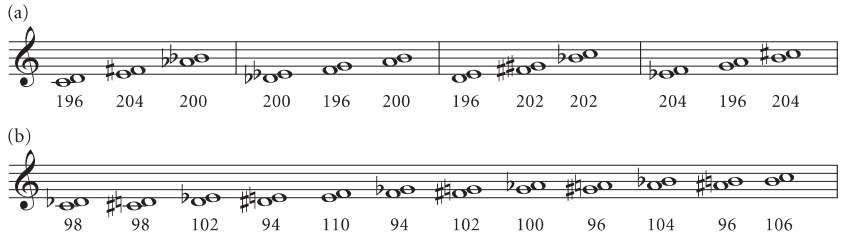
Why should it matter to have distinctive diatonic scales? The perceptible dimensionality of the music is multiplied. The playing of a keyboard fugue is the simultaneous control of three or four melodic shapes, and 'counterpoint' and 'harmony' are the complex interactions of those shapes. Modulation comes from the introduction of foreign notes (irregularities) into a melody's prevailing scale: the listener's mind notices the irritation and assigns the anomalous notes to whatever competing scale best contains them. Is that not the way basic human perception of language works: noticing irregularities in the flow of sound, and parsing the stream of input into meaningful packets by analogy with already known patterns?[52]

The mind is very good at dealing with unexpected and seemingly irrelevant stimuli, such as the mention of strawberries. When all the scales have distinctive aural signatures, as in Bach's temperament, the listener's task of parsing the music is much easier (especially when receiving several contrapuntal voices simultaneously). Furthermore, the mind is continually challenged by the dynamics of all this: attention is captured and maintained. Passive voice, active, tart strawberry, fragments, unorthograffy Violation of Expectations, just the right balance of asymmetric contrasts and flow and purposeful irritation. In summary: *when heard in a carefully balanced unequal temperament, the music is much more transparent, interesting, and engaging, all the way through*.

Ex.1 shows all tones ('whole steps') and semitones ('half steps') of Bach's temperament, measured in cents.[53] In my notation here, the notes are to be played *in succession* either ascending or descending, not crushed together simultaneously. The measurement here is of *melodic* quality.

There are four different sizes of tones and eight different sizes of semitones. That is: these linear intervals have a perceptibly different sound from one another, providing expressive inflections within melodies. At the same time, the differences are subtle enough that they do not draw undue attention or ruin melodic smoothness: listeners may choose to notice or ignore them as they wish.[54] The differences are most noticeable when listening very closely at a harpsichord, in isolation, playing the notes slowly.

Ex.1 Intervals in Bach's temperament, measured melodically in cents: (a) tones; (b) semitones



Press one note, hold it long enough to establish it in the ear as home base, then move to the neighbouring note.

Tones of size 196 are characteristic of regular ⅙ comma; of 204 from Pythagorean intonation (generated by pure 5ths); and 200 from equal temperament. The 200 of A–B is a coincidence from the raising of B, and the 202 of F♯–G♯ and B♭–C arise from the crossover points from one type of 5th into another.

In table 2 these steps are arranged into the four- and six-note subsets of the major and minor scales. The chart therefore describes all '*ut re mi*' and '*re mi fa*' combinations in Bach's temperament. In addition to being higher or lower in pitch, every scale sounds *absolutely distinct* from every other scale, due to the different sizes and relationships of their intervals. The effect, or one might even say 'personality', of every musical key becomes recognizable. Compositions inherit this character from the home key (tonality) and any other major or minor keys the composer visits as the music moves along from section to section.

A tetrachord is a set of four rising notes in a major scale: *ut, re, mi, fa*. One can divide the complete scale

Table 2 Scale (tetrachord and hexachord) analysis of all available tones and semitones in Bach's temperament, measured melodically in cents (a) *ut re mi* (tertia major); major scale: *ut–re–mi–fa, ut–re–mi–fa*

Tetrachord	*ut–re*	*re–mi*	*mi–fa*	*(ut–re) fa–sol*	*(re–mi) sol–la*	*Hexachord (major)*
C–D–E–F	196	196	110	196	196	C–D–E–F–G–A
G–A–B–C	196	200	106	196	196	G–A–B–C–D–E
D–E–F♯–G	196	204	102	196	200	D–E–F♯–G–A–B
A–B–C♯–D	200	204	98	196	204	A–B–C♯–D–E–F♯
E–F♯–G♯–A	204	202	96	200	204	E–F♯–G♯–A–B–C♯
B–C♯–D♯–E	204	200	94	204	202	B–C♯–D♯–E–F♯–G♯
F♯–G♯–A♯–B	202	200	96	204	200	F♯–G♯–A♯–B–C♯–D♯
D♭–E♭–F–G♭	200	204	94	202	200	D♭–E♭–F–G♭–A♭–B♭
A♭–B♭–C–D♭	200	202	98	200	204	A♭–B♭–C–D♭–E♭–F
E♭–F–G–A♭	204	196	100	200	202	E♭–F–G–A♭–B♭–C
B♭–C–D–E♭	202	196	102	204	196	B♭–C–D–E♭–F–G
F–G–A–B♭	196	196	104	202	196	F–G–A–B♭–C–D

(b) *re mi fa* (tertia minor); Dorian mode: *re–mi–fa–ut, re–mi–fa–ut*

Tetrachord	*re–mi*	*(mi–ut) mi–fa*	*ut–re*	*re–mi*	*(5th–min 6th) mi–fa*	*Hexachord (minor)*
D–E–F–G	196	110	196	196	104	D–E–F–G–A–B♭
A–B–C–D	200	106	196	196	110	A–B–C–D–E–F
E–F♯–G–A	204	102	196	200	106	E–F♯–G–A–B–C
B–C♯–D–E	204	98	196	204	102	B–C♯–D–E–F♯–G
F♯–G♯–A–B	202	96	200	204	98	F♯–G♯–A–B–C♯–D
C♯–D♯–E–F♯	200	94	204	202	96	C♯–D♯–E–F♯–G♯–A
G♯–A♯–B–C♯	200	96	204	200	94	G♯–A♯–B–C♯–D♯–E
E♭–F–G♭–A♭	204	94	202	200	96	E♭–F–G♭–A♭–B♭–C♭
B♭–C–D♭–E♭	202	98	200	204	94	B♭–C–D♭–E♭–F–G♭
F–G–A♭–B♭	196	100	200	202	98	F–G–A♭–B♭–C–D♭
C–D–E♭–F	196	102	204	196	100	C–D–E♭–F–G–A♭
G–A–B♭–C	196	104	202	196	102	G–A–B♭–C–D–E♭

of eight notes into two halves: 'lower' and 'upper' tetrachords of four notes each:[55] *ut–re–mi–fa*; *ut–re–mi–fa* (e.g. C–D–E–F; G–A–B–C). The upper tetrachord G–A–B–C then begins the next scale, continuing D–E–F♯–G. The F♯ is the new note, and it gets a new (raised) key lever on the keyboard as we already have an F. The new note each time is the *mi* of the upper tetrachord, tuned (as some size of fifth) from the already available *mi* of the lower tetrachord.[56]

The older system of hexachords uses the set of six rising notes: *ut, re, mi, fa, sol, la.*[57] Like a complete major scale, a major hexachord (natural hexachord, *hexachordum naturale*) can also be broken down into halves: as '*ut re mi*' twice in succession. Similarly, a minor hexachord (soft hexachord, *hexachordum molle*) has '*re mi fa*' twice in succession. These two groups of three notes are mentioned in Bach's introduction to *WTC*: 'The Well-Tempered Clavier, or, preludes and fugues through all the tones and semitones, both as regards the *tertia major* or *ut re mi* and as concerns the *tertia minor* or *re mi fa*.'

The only symmetric major hexachord in Bach's temperament is C–D–E–F–G–A. Its symmetry comes from the regular naturals: the home key of C major, '*ut re mi*'. The most nearly symmetric minor hexachords are similarly in the simplest '*re mi fa*' minor keys: D–E–F–G–A–B♭ and A–B–C–D–E–F.[58] That is: the keys with fewest accidentals are melodically the smoothest, having tones of equal size.

The most remarkable features in table 2 are the columns '*re–mi*' and '*mi–fa*' shared by the major and minor tetrachords. The lower and upper tetrachords to assemble each scale are distinct, in the paired qualities of those two melodic intervals '*re–mi*' followed by '*mi–fa*'. Listeners can therefore recognize any tonality immediately by its subtle '*re–mi–fa*' inflections, and hear an objective difference whenever the music modulates.[59]

Taking a step out to the big picture, to show the importance of this observation: all the scales are *equally usable* in musical quality, yet distinct. Transposition of any theme, any section, any entire piece causes a profound change in the relationships of all the intervals; an audible change of character in the music. There are exponentially increased musical resources available, compared with the assumption of equal temperament upon which so much of modern music theory is based. Change the melodic sizes of steps and leaps from a constant to a subtly variable parameter, and everything becomes fluid: more challenging to control analytically, but also more exciting. It is a world of sound-relationships, each with particular and recognizable identity. Pieces of tonal music ebb and flow organically, through their modulations of character.

Tonal theory will have to catch up with this, supplementing approximately 250 years of simplification. The choice of temperament (and this is true of all 12-note unequal temperaments, not only Bach's) is *not* independent of the content of a piece of music, at least where keyboards are concerned. The field of music theory has typically kept intervallic analysis and tuning details apart for convenience; all three-note and four-note chords with a particular pattern of tones and semitones are treated as basically equivalent, regardless of key centre. The little-examined assumption behind this is the consistency of equal temperament (and behind that, the broader collection of regular 'mean-tone' layouts with their easily predictable equivalences and transposition patterns).

As Bach's temperament (along with some others) makes clear, melodic motion from C♯ to D is *not* equivalent to motion from D to E♭, even though in both cases we are dealing with correctly spelled diatonic semitones, and even if we shifted the frequencies of everything so the starting pitches were identical. Those semitones are perceptibly different sizes, and that fact is a musically expressive resource. For example, transpose the Sarabandes of the suites BWV823 and 819 to various different keys; the music loses much of its poignancy and in compensation it gains other characters altogether, depending on what key is chosen.

Such things can also be heard directly, with a chord in isolation as a representative exercise. C–D–F–B and E♭–F–A♭–D do not sound at all alike in Bach's temperament; the former is gentle and the latter is piquant. E–F♯–A–D♯ is brisk and crisp, while F♯–G♯–B–E♯ seems both delicate and clean. Other listeners will come up with different words here, akin to the descriptions of competing wines, but the point is that all these equivalently spaced chords sound radically different from one another.

Absolute placement within the complex set of 12 notes matters, absolutely! Even more complex chords than that, generated by linear passing motion or ornamentation, are delightfully rich for study: a good place to start is the modally ambiguous material in *Clavierübung III*, tuned correctly.

Major 3rds, minor 3rds and mean accidentals

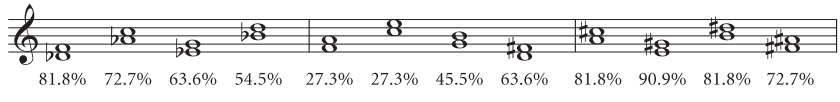
The 12 major 3rds have a steady progression around the circle of 5ths: with C–E and F–A being the most nearly in tune, to E–G♯ as the farthest out of tune. See ex.2 and fig.1. (Review also the *Chorton* graph, fig.3 in part 1.)

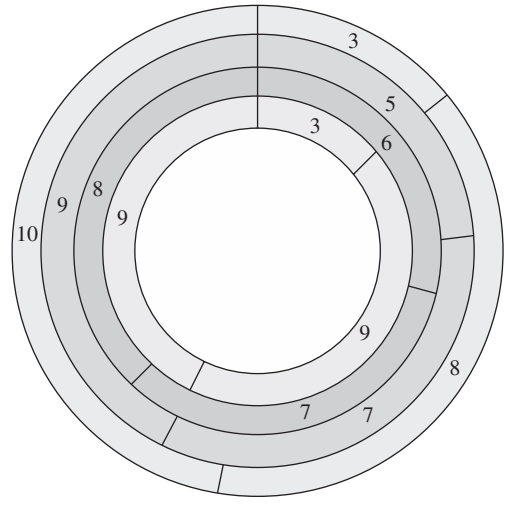
In effect, the music sounds more mellow or relaxed when in the major keys of the fewest sharps or flats, and brighter or more tense in the key area around E major. That is because of the varying beat in the major 3rds, like a tastefully graded vibrato.

Arranging all four sets of major 3rds as they are stacked into octaves, the proportions are as shown in fig.1 (and at the bottom of Table 1 in part 1). The C–E group has the strongest contrasts, and the B♭–D group the gentlest.[60] It is clear that the simplest keys are favoured as the best in tune, having the most repose (the slowest vibrato). The favouring of intervals with flats also becomes obvious, at the expense of the sharps. To restate that same phenomenon differently: the sharps gain brilliance and restlessness when used in major triads, each needing to resolve as a dominant into the next tonic.

E major was traditionally the edge of the meantone 'universe' in the 17th century. We have trained ourselves since Bach's lifetime to expect any motion beyond that to continue in the same direction, as most circulating 'workmaster' temperaments indeed do: with the extremity moved out to F♯, D♭ or A♭ major, and an axis of best-to-worst, moving steadily from C to F♯ major.[61] But, that is our expectation, not necessarily Bach's. His temperament provides instead a gentle slope back down

Ex.2 Qualities of the major 3rds in Bach's temperament. The number with each interval indicates the sharpness (being wide of a pure 5/4 ratio), as percentage of the SC.





Major 3rd sharpness
(1/21 portions of diesis)

C–E	3
A♭–C	8
E–G#	10
G–B	5
E♭–G	7
B–D#	9
B♭–D	6
D–F#	7
F#–A#	8
F–A	3
A–C#	9
D♭–F	9

Fig.1 Graph of the major 3rd qualities, Bach's temperament.

through the extreme sharps into the flats. E major is a mountain-top in which the dominant and subdominant triads are both better in tune than the tonic is. And as Bach's music uses B, F♯, D♭ and A♭ major triads so often in his harmonic progressions, the benefit of his own temperament (where those are relatively calm, yet still colourful) is enormous.

The balances within minor tonalities are remarkable. A minor has the blandest and most stable tonic scale and triad, but the strongest and most restless dominant. In the context of A and D minor, melodically, the notes D♯ and G♯ sound especially 'hard' and grab a listener's attention. Around the circle of minor keys, as the tonic develops more character its dominant softens. E, B and F♯ minors all have sweet, not harsh, dominants.[62] At the end of the traditional mean-tone universe in the other direction, the spicy key of F minor, the dominant is the gentlest C major. The more extreme flat minors (beyond F) are dark and suspenseful, while the extreme sharp minors (beyond F♯) are reedy and poignant.

Bach's drawing (illus.1 in part 1, and its web supplement) appears to have additional calligraphic games in it. Look again at the word 'Das', with its unusual capital. The beginning of the stroke forms a C and the end of it forms an E, looking also like the word *Es* (the German name for E♭). The D is between them, reading along that stroke. Does not this single capital D illustrate *C–D–Es*, i.e. '*re mi fa*' of C minor? And, the W of *wohltemperirt* has some funny business in it, as well, looking like a capital E. Taken with the D of '*Das*', we have *C–D–E*, i.e. '*ut re mi*'. The capital P of *Praeludia*, when viewed upside-down and in a mirror, looks like a dismantled treble clef. The flourish at the bottom of the page appears to have something to do with three spiralling gestures followed by five side-to-side swishes, which might have additional meaning for tuning or something else. Not to belabour this, I believe that the page's layout and calligraphy deserve much closer study than they have received. Obviously Bach spent plenty of care on the layout of his words and other strokes.[63]

In that word 'Das', the *Es* of the capital D is bisected by the long vertical stroke, and two dots are placed equidistant on either side. Does this have anything to do with minor 3rds? Indeed it might. In Bach's temperament four of the five accidentals, including *Es*, are placed exactly where they bisect tritones.

C–E♭–F♯–A: three minor 3rds each 81.8 per cent SC too narrow; that is, C–E♭–F♯–A are spaced equidistantly amongst themselves, and the E♭ and F♯ therefore both split tritones exactly in half.

G–B♭–C♯: two minor 3rds of 72.7 per cent SC narrow, with B♭ splitting G–C♯ exactly.

F–A♭–B: two minor 3rds of 90.9 per cent SC narrow.

The exceptional note is C♯/D♭, which is mean as a *major* 3rd between A below and F above (each of these two major 3rds being 81.8 per cent SC too wide). C♯/D♭ is also mean as a semitone between C and D: 98 cents each.

The important observation here is: all five of the keyboard's raised notes are tuned *mean* (i.e. at the exact geometric average) within some wider interval(s). That gives them flexibility as enharmonic equivalents. This layout gives plenty of symmetry to minor 3rds, and to diminished triads. Because those five 'accidentals' are all at *average* positions, they also serve easily as pivot notes for modulations.

Musical illustrations of Bach's temperament in *WTC*

We have dissected the temperament's innards; let us now put it back to its musical context to watch it dance.

Art is not merely the avoidance of error—dodging any hazardous intervals or chords—but the positive demonstration of significant truths. Bach illustrated the unique properties of his temperament directly in the *sound* of the music (working in his own best medium of expression), especially in the first book of *WTC*. This is even more spectacular a result than the title-page's presentation of the temperament in isolation.

First, ex.3 gives a brief review of the temperament's symmetries, as we will see many of these in the music; these are all *musical* effects noticeable by listening closely to the keyboard.

Now, the music, to see what Bach made of these. Here are some features that I have noticed, usually as early as the opening bars,[64] when playing through the *WTC* on harpsichord and clavichord

and listening very closely to the intervals. The musical subjects themselves appear, in large part, to be inspired by those irregularities and symmetries in the temperament.

The C major prelude celebrates 5 + 3 (the basic pattern of Bach's diagram), and introduces every pitch we will hear in the book. The five accidentals each show their individual colours against the neutral background of C major. The fugue subject then highlights all six notes of the regular subset: the hexachord C–D–E–F–G–A. Its first half is the rising tetrachord, and the real answer in G quickly gives us the other half of the C major scale: already demonstrating the different character of that second tetrachord (with its sharpened *mi*, B). The wide spacing of the voices gives an interesting moment at the end of bar 23, setting up the pedal point: due to the way B is tuned especially high in context of the other naturals, the 4–3 suspension over G seems like a resolution from consonance into dissonance.[65]

The C♯ major fugue (and the A♭ major fugue) emphasizes the melodic G♯–E♯ interval (A♭–F) that is poor in other temperaments (much too wide), but fine here: perhaps serving as a test piece for the temperament's accuracy.

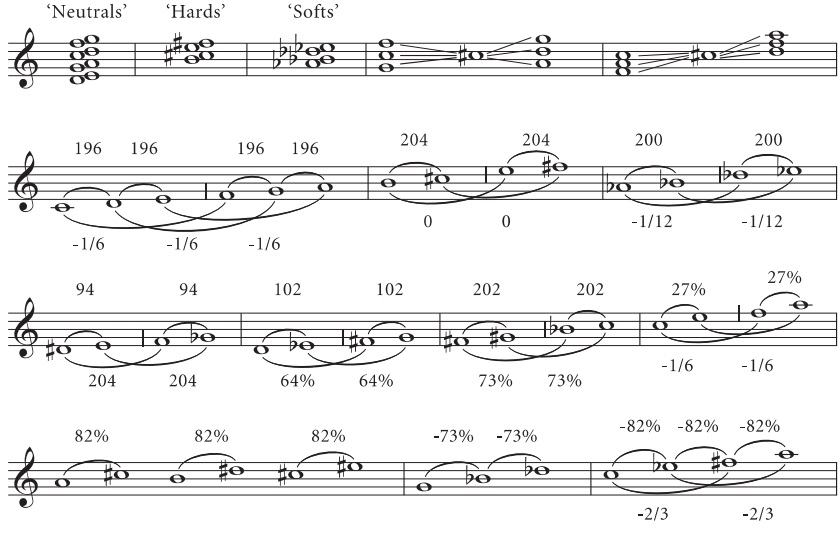
In the C♯ minor fugue, the melodic diminished 4th of the subject (B♯–E) is the smallest such interval available, being tuned as the best major 3rd. This fugue (unusually) has real entrances in *six* different keys, displaying all the following diminished 4ths: B♯–E, F𝄪–B, E♯–A, C𝄪–F♯, D♯–G and A♯–D. Those are all the six positions in Bach's temperament where that interval is narrower than or equal to its size in equal temperament.

The D major and minor fugues both explore the melodic quality of 6ths: B, F♯ and B♭, the first notes to be tempered outside the regular 55-division in either direction.

The E♭ major prelude presents the subset of equal-tempered notes directly in the first bar: E♭, G, A♭, B♭ and D♭. In the fugue subject the leap of a 7th from F to E♭ is a *pure* 16:9 interval: a startling sound in context where everything else is tempered.

The E♭ minor prelude at its climax in bar 26 lands on the brightest (most out of tune) chord in the entire temperament: F♭ major. The D♯ minor fugue has both a delicacy and a penetrating strength, a sound that seems to have been refined by fire. The *stretti* and augmentations ring out: time itself is moving at several speeds at once. By comparison, if the fugue is

Ex.3 Symmetries within Bach's temperament. The main axes of symmetry are the note C♯ (tuned mean within many wider intervals), and the crack between E and F. The naturals (playing the set D–E–G–A–C–D–F–G, all together) sound like handfuls of 'neutrals', by contrast with the 'hards' (B–C♯–E–F♯) and the 'softs' (A♭–B♭–D♭–E♭).



played in its original key of D minor, it merely seems ordinary and archaic. The tragic intensity is softened.

The E major and B major fugue subjects both play games with all four notes of their 'Pythagorean' subset: B, C♯, E and F♯.

The E minor fugue showcases the chromatic scale descending from E: as D♯–E is the smallest semitone in the temperament, and E is the last of the *regular* notes.

The F minor and B minor fugues both explore the melodic implications of multiple sizes of semitones. I believe Bach is illustrating the subtle inflections of these intervals, trying to give us a comprehensive view of the expressive possibilities. The end of the B minor subject further emphasizes the only point in the temperament where an exact mean semitone is available: C♯, between C and D.

The F minor, G minor and B♭ minor preludes all have plenty of occurrences of A♭–C and D♭–F, the two intervals that are worst in most other temperaments; these are the spots that force Bach's temperament to be used, unless the performer talks himself/herself and the audience into accepting harsh 3rds and 10ths.[66]

The F♯ minor fugue is full of semitone suspensions and appoggiaturas in exposed situations; the exploration of their melodic quality seems to be a paramount point of the piece. E♯–F♯ and D♯–E are the smallest semitones available in the temperament, at 94 cents each; followed by G♯–A and A♯–B at 96 each.[67] The F♯ minor subject here presents the two 96-cent semitones plus B♯–C♯ (98). Its real answer in C♯ minor gives us the two 94-cent semitones plus F𝄪-G♯ (100). The two inverted statements, at bars 20 and 32, give us one of each in descending position, for contrast. Along with these, Bach has worked the temperament's remaining smallish semitone (C♯–D at 98 cents) prominently into the episodes based on the countersubject.[68] The broader point is, apparently: small semitones are a beautifully expressive resource, as long as they do not get much smaller than this![69] Furthermore, this fugue loses its poignant character if played in other keys (for example, in either F minor or G minor, in both of which it seems merely bland), or if played in equal temperament.

In the G major fugue the two most attention-grabbing features of the subject are the rising 7ths D–C and F♯–E. This juxtaposes the different melodic sounds of normal ⅙ comma and the pure 16:9 ratio.

The G♯ minor prelude illustrates the lowest of all 6th degrees, melodically (E to D♯, 94 cents). Notice also the emphasis on the C♯–E minor 3rd near the end, the smallest minor 3rd available in the temperament (the only one that is 100 per cent SC narrow from a pure 6:5 minor 3rd). The G♯ minor fugue starts with the only available equal-tempered semitone. The B♭ minor fugue, like the G♯ minor prelude, emphasizes the other 94-cent (smallest) semitone: again approached by a large leap.

The A♭ prelude begins audaciously with tonic triads in various inversions: which works only in a temperament favourable to this notorious and formerly forbidden key. The fugue similarly emphasizes the intervals A♭–C and D♭–F, which are the harshest major 3rds in other circulating temperaments but euphonious in Bach's. What better way is there to test a good temperament than to play this prelude and fugue? Later, the A♭ major and G♯ minor pieces of the second book give an even more thorough workout of the temperament—as if these are more test pieces for tuning accuracy.[70]

The A major fugue subject (along with its real and tonal answers) is an odd melody: it looks like a tuning test for the quality of 3rds and 4ths, and with Bach's temperament it displays all three types of 4ths: pure, 1/12 and ⅙ comma.

The A minor fugue juxtaposes the calm E–D–C–D–E with the stark (and widest) G♯–E. The concentrated level of dissonance from all the 'hot' sharps continues until the breather at bar 40. The accented parallel 5ths in bar 55 (B–F♯ pure, then C–G tempered) plunge us back into the maelstrom.

The B♭ minor fugue's subject and tonal answer give us the single odd 5th/4th in the temperament: B♭–F, narrow as a 4th and wide as a 5th. The F–G♭ minor ninth in the subject is poignant as the smallest one available. At bar 37 the grand cadence into A♭ major is of course fine in Bach's temperament, but horrible in most other temperaments. Likewise, the numerous prominent placements of the D♭–F major 3rd on top of the texture look like an exercise in making other temperaments fail.

The B minor prelude has a rising Dorian scale in the bass, and this is the only key in which the Dorian scale presents all four available sizes of tones. Meanwhile, the right hand starts phrases with all three of the available pure 4ths: F♯–B and C♯–F♯ in bar 1, and B–E in bar 6.

Similarly: Bach's chorale preludes may be treasure troves, as to any theological commentary he might make in response to the hymn texts. And, if this temperament was used regularly in accompaniment of the vocal works (see discussion in part 1), with its subtleties of melody and harmony, the possibilities of rhetorical gesture are even more staggering than we may have suspected.

The *Early music* website provides three further case studies from Bach's keyboard repertory: *O Mensch, bewein dein' Sünde groß*, BWV622; *Clavierübung III*; and *Kleines harmonisches Labyrinth*, BWV591. These raise further related issues about melodic shape, accentuation, the purpose of the four *Duetti*, the *Affekt* of chorales, and enharmonic handling.

Conclusions

This discovery of Bach's handwritten instructions from 1722 is not restricted to application in that single book of *WTC*, or its sequel in the 1740s. As a practical tuning for keyboards, it offers musical advantages over other temperaments commonly employed for *any* of his music. It works perfectly in the solo and ensemble music from all parts of his career, both instrumental and vocal. Therefore, the tentative conclusion is attractive: that Bach indeed intended it to be used in that capacity as his all-purpose solution.

A caveat for ensemble work with this and any other previously unfamiliar temperament: allow plenty of rehearsal time consistently with it, to learn the shapes and tensions—especially the expressivity of the sharps. Temperaments are not merely skins to apply to already finished interpretations. They can guide and reshape the whole.

Since this April 2004 discovery I have discussed with dozens of colleagues aspects of Bach's tuning and related issues. The process of cross-checking and confirmation in the practical and theoretical avenues has been extraordinarily fruitful. At first I did not expect this investigation to have much if anything to do with the organ works, or the ensemble music, or the transposing *Chorton/Cammerton* situation (see part 1); but it all fell into place, and has solved *all* these outstanding problems of the repertory. I remain as astonished as anyone. The resonance of the instruments is qualitatively different from anything we have heard from them in any other tunings, even in the little-known and excellent 'Bendeler III' of 1690,[71] and Sorge 1758 (see the website appendix for each): two other brilliant temperaments that deserve further use.

After several months hearing my play-throughs of Bach's repertory, my wife as a listener offered the remark: 'But it just sounds right all the time, and it's *unremarkable*. All the other ways you used to tune, and all our recordings, make the harpsichords sound tinny and harsh, and hard to listen to.' Indeed; all music works so equally well that the lack of problem is unremarkable. In its brilliant highlighting of the music, the temperament does not call attention *to itself* (nothing ever sticks out to sound 'wrong'), and that is probably why Bach and his contemporaries had so little to say in extant sources. It comes across as 'equal' temperament in which everything is fine all the time, and *that* is what the 18th-century witnesses described: as a contrast against the bumpiness and modulatory restrictions of regular (mean-tone) temperaments. The remarkable thing here is that Bach's pattern was ever *lost*; after all, he wrote it in the most obvious place he could have put it, at the top of the title-page of a book about tuning.

A tacit assumption in music history and criticism is that most of music's meaning is available to us in words. Programme notes for recordings and concerts tell us what we 'should' be listening for and noticing. We study the culture in and around the compositions, any extra-musical subjects and associations, names, personalities and influences. Dramatic works have meaning in their plots and poetry. Again, it's the words. Bach's meaning may be largely elsewhere, however, as his vocal works and organ chorales reveal. The associated words and imagery are primarily the scaffolding for the building of additional commentary in *sound*.

Bach's instrumental works are invested with meaning as well, through his inventively irregular

structures and his blending of arts. David Yearsley in *Bach and the meanings of counterpoint* (2002) and Laurence Dreyfus in *Bach and the patterns of invention* (1996) have provided exemplary essays of analytical depth: exploring the cross-fertilization of ideas and influences, showing Bach to be in absolute control of his materials. However, the dimension of Bach's specific intonation has been (necessarily) lacking from such presentations, simply because it was not yet known. I have proposed here that Bach's rich meaning resides also—inextricably—in that subtly irregular intonation: with evidence not only in notes (and our aesthetic appraisal of them) but also in a gestural design he drew on paper. The correct tuning of the intervals according to his expectations reveals this lost layer of his art, perhaps making the spiritual content of his music more easily perceptible and measurable (showing his craftsmanship of *Affekt* to be both specific and objective). We return to Bach's music so often already because the *sound* of it moves us; and now even the sound itself is shown to be different from our modern expectations. It too is a blend of arts in an uncommon way. There is nothing ordinary or average about it.

After months of listening to Bach's music and other repertory in his tuning, how do the previously familiar tuning methods (Vallotti, Werckmeister III, Kellner, Barnes, Neidhardts, regular and modified mean-tones) sound to me? In a *word*, 'unsatisfactory'! In music with sharps, I find the sound pleasant but bland, lacking the heat and colourful tension, while the too-strong dominant triads of B and F♯ major are obtrusive in musical context. And in music with flats, the occurrences of the notes A♭, D♭ and G♭ stick out, and melodies seem lopsided. Although equal temperament avoids the problems of grossly wrong accidentals, it also comes across as dull: lacking any asymmetry, it does not hold a listener's attention by itself, as Bach's does. My own performance style on harpsichord and organ has become simpler and more 'right-brained', as I listen and react to the directions the notes are already taking due to their intonation. I feel less need to 'interpret' the music in projecting it. The exercise of physically tuning and playing with Bach's temperament has heightened my sensitivity to pitch, especially in melody. I find it more difficult now to enjoy recordings that I have loved for years, since their out-of-tune moments seem so arbitrary and jarring.

Pandora's box is open. Other tonal music than Bach's also sounds stronger now, so warm and full of character. I feel that I finally begin to understand the well-known remark that no one else could tune to Bach's satisfaction.[72] With no wish to be too pedantic or dogmatically restrictive, I urge readers to set Bach's temperament and hear it directly in musical context, as no article can do full justice to its wonders. The meaning is in the *sound*.

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1 Mark Lindley, *Lutes, viols and temperaments* (Cambridge, 1984), pp.3–4.

2 Tuning instructions are in part 1 and at www.larips.com.

3 The 1715 edition of *Ariadne musica* has various spirals and flourishes on both its title-page and interspersed among the pieces; one on the title-page (attached to the name 'Fischer') is exactly the same as Bach's, when turned upside-down. Friedrich Suppig's 1722 *Labyrinthus musicus* and *Calculus musicus* also both have spirals on their title-pages. I believe that these spirals are references to the view of tuning as a progressive spiralling maze through the keys: the Ariadne legend.

4 *The New Bach reader*, ed. H.T. David, A. Mendel and C. Wolff (New York, 1998), no.306; *Bach-Dokumente*, iii (1972), no.666.

5 Bach used them especially in Phrygian, Neapolitan and subdominant harmonic contexts; and occasionally as tonics.

6 These are the two intervals that stand out most readily as 'wrong' in all the temperaments derived from Werckmeister's use of pure 5ths around the flat side.

7 Many musicians and theorists had strong opinions about this, as is made clear in R. Steblin, *A history of key characteristics in the eighteenth and early nineteenth centuries* (Rochester, NY, 2/2002).

8 When the relative qualities of its major 3rds are graphed, and compared against the graphs of other temperaments, the shapes are

not similar. To my knowledge, the only 18th-century temperament with the same harmonic shape as Bach's is Sorge's from 1758, discussed in part 1 and below.

9 Dedicated to Bach's brother; perhaps a tribute to the household where he learned to tune in such a way that there are no real limits beyond the traditional dead-end of E major? This piece appears designed to show everything E major can do.

10 There is also a source of the BWV542 fugue (only) in F minor rather than G minor.

11 Before 1717, according to the 1998 edition of *BWV*; and see the discussion in P. Williams, *The organ music of J. S. Bach* (Cambridge, 2/2003), pp.159–62.

12 M. Lindley, 'J. S. Bach's tuning', *Musical times*, cxxvi (Dec 1985), presents the Neidhardt and Sorge tuning philosophies, remarking that in the *WTC* 'the different keys are in fact treated differently. . . A tuner who can accommodate and heighten these contrasts while keeping the extreme keys from sounding sour on the instrument to hand will probably dismiss the argument that *Das wohltemperirte Clavier* "must be performed with an equal-tempered keyboard".' The article concludes with a comment about the six-part ricercar of *Das musikalisches Opfer*, 'which when played on the organ sounds better (I believe) in equal temperament than in an historically appropriate unequal one'. The ricercar's main problems in other temperaments are in the intervals D♭–F and A♭–C, and the notes C♭ and G♭; but Bach's temperament now solves all that, dismissing this assertion about equal temperament. The 'most historically appropriate unequal' temperament is surely Bach's own.

13 Other irregular circulating temperaments also bring out 'colour' changes when Bach's music modulates, but they are not as smooth as this: like a colour reproduction of a painting, but printed with a distractingly incorrect balance of the pigments, changing the overall effect of the work. In the same analogy, equal temperament is merely a black-and-white print from a colour painting: with the contrasts washed down to shades of gray.

14 BWV922 is from 1714 at the latest, according to the 1998 edition of *BWV*.

15 Sources of BWV910 also exist in F minor rather than F♯ minor. Another clue to a possible origin in F minor is the section of bars 108–34: if played in F minor on a keyboard in regular mean-tone (⅙, ⅕ or ¼), this passage explores all the extremes completely (like the improvisatory 'noodling' of a young keyboard player trying out all the sounds of the standard temperament).

16 BWV910 is especially the one that has led to retunings (or the use of equal temperament) in recordings of these toccatas; and I have played this toccata for several years transposed into F minor to dodge the problems other temperaments present. Now this will be no longer necessary, as it works fine as notated in F♯.

17 R. L. Marshall, 'Organ or "Klavier"? Instrumental prescriptions in the sources of Bach's keyboard works', in both his own book *The music of Johann*

Sebastian Bach: the sources, the style, the significance (New York, 1989) and in *J. S. Bach as organist*, ed. G.B. Stauffer and E. May (London, 1986). I have been playing these toccatas on both harpsichord and organ for 20 years, anyway, from Heinz Lohmann's Breitkopf edition: they are terrific organ pieces.

18 Discovered in 1985; the *NBA* edition is Bärenreiter 5181, ed. C. Wolff (1986). See especially the chorales BWV1093, 714, 742, 1108, 1110 and 1113.

19 M. Rathey, 'Die Temperierung der Divi Blasii-Orgel in Mühlhausen', *Bach-Jahrbuch*, lxxxvii (2001), describes Bach's supervision of this organ's renovation in 1708, and offers a circumstantial guess that the new temperament installed was Werckmeister III. However, Bach's contemporary compositions (especially BWV729, 549 and 575) strain the limits of 'barbarism' in Werckmeister's temperament, and argue that young Bach already had something much better than that.

20 This appears to be a general characteristic of any pieces that are in D minor or A minor, especially: as each accidental is a different amount of tempering away from its expected position in the 55-division, each gives a distinctive profile against the background of regularity. The same effect emerges from the playing of 17th-century repertory that is mostly diatonic on the naturals, for example the fantasias, canzonas and ricercars by Froberger. In the old-fashioned sense of hexachord mutation: as every hexachord here sounds *distinct*, the mutations all become easily perceptible and keep the music ever new as it moves along.

21 These are the two Bach books that have traditionally 'required' equal temperament: until the discovery of Bach's temperament, which interacts with their contents in extraordinary ways.

22 E♭ major = heaven, and C minor = earth, respectively?

23 These are 'hidden' messages in the *sound* of the intervals in Bach's temperament, perhaps for the delectation of the *Kenner* to whom

C*lavierübung III* is dedicated, as opposed to the *Liebhaber* of the other books. Players and theorists who understood the practical motions of Bach's temperament, and its deep implications for the music, were probably as rare then as now. And at Frederick's court the keyboardist (C. P. E. Bach) was perhaps the only person who would catch a secret in the *Canon per tonos*: this piece celebrating the king's ascendancy gives a mixed and possibly volatile message, in the sequence of contrasting tonalities it explores.

24 Frederick the Great (1712–86), employer of C. P. E. Bach and dedicatee of *Das musikalisches Opfer*.

25 BWV645–50, 651–68.

26 Williams, *The organ music of J. S. Bach*, pp.111–18.

27 And, more recently, on Goshen College's organ, Taylor & Boody Opus 41. The recording will be available soon from www.larips.com and www.goshen.edu.

28 How did Couperin's music sound at Bach's house, and what was in their letters of personal correspondence?

29 The notes C♯/D♭, E and G are pivot notes belonging to more than one subset.

30 Tunings generated by 5ths of consistent size: which also put tones as the geometric mean within major 3rds, and are therefore 'mean-tone' temperaments.

31 Lest I be accused of pseudo-mysticism, I shall not press that coincidence further.

32 Such semitones are characterized as 'too dull for Bach's music' in M. Lindley, 'A suggested improvement for the Fisk Organ at Stanford', *Performance practice review*, i (1988). The article is a concentrated explication of basic temperament theory and Neidhardt's priorities of the 1720s, with circumstantial connections to Bach. Lindley's own second temperament suggestion in his fig.15b approaches Bach's solution fairly closely, except for the treatment of E–B and his insistence that the D♭ major area should be the least in tune.

33 A diatonic semitone is one where there is a change of note name, e.g. E to F. A chromatic semitone (colour change, 'chromatic') has no change of note name, but the two pitches are aspects of the same note, e.g. B♭ to B. This distinction is most obvious in the stronger variations of mean-tone such as quarter- and ⅓-comma.

34 Listeners who have either a synaesthesia or absolute pitch can perceive additional characteristics of the keys, even in equal temperament; but those phenomena arise from complexities in human perception, not from more objectively measurable ratios and relationships of frequencies. The distinctive key characters in Bach's and other irregular temperaments arise from the relative tuning of various intervals, apart from any considerations of human perception.

35 See Lindley, *Lutes, viols and temperaments*, pp.23ff.

36 The *Bach-Dokumente* editors, Wolff, Yearsley and others have speculated variously about the identity of this mysterious FABER, usually tying it to people named Faber or Schmidt whom Bach knew in the 1740s. But why could the piece not be simply a game or gift for a Bach family member, perhaps Nikolaus (who had initiated the musical use of 'B–A–C–H' years earlier); and a celebration of family and life cycles in general? As the piece renews itself on each repetition, so does each new year.

37 Francesco Tosi, *Observations on the Florid Song* (1723), trans. Johann Ernst Galliard (1742); first two chapters.

38 This works out as almost exactly equivalent to regular ⅙ comma mean-tone, in practice. It is a clever 18th-century way to summarize the basic diatonic and chromatic tuning issues, keeping things to simple integers for the understanding of common musicians.

39 For explanation and background, see especially B. Haynes, 'Beyond temperament: non-keyboard intonation in the 17th and 18th centuries', *Early music,* xix (1991), pp.357–81, and J. H. Chesnut, 'Mozart's teaching of intonation', *Journal of the American Musicological Society*, xxx (1977), pp.254–71. Musicians and theorists who endorsed or commented on this include Sauveur (1707), Tosi (1723), Telemann (1742/3) and Sorge (1748). The many keyboard temperaments of Neidhardt, with ⅙ comma 5ths and 1/12 comma adjustments to them, confirm this as well.

40 Johann Joachim Quantz, *Versuch einer Anweisung die Flöte traversiere zu spielen*, trans. E. R. Reilly as *On playing the flute* (Boston, MA, 2/1985).

41 This, to me, is corroboration that Bach's temperament or a similar one was still in use by the court keyboardist (C. P. E. Bach) in 1752. According to Quantz: string players tune open strings to the keyboard's regular ⅙ comma naturals. The keyboard's accidentals are all at compromised positions within 'subsemitones' such as G♯/A♭ (which are one comma apart from each other). The keyboard accompanist must take special care to voice low (or perhaps leave out altogether!) the following specific notes, in exposed texture above the bass: G♯, D♯, A♯ or E♯ as the top of major thirds, or C♭, D♭ or E♭ as the top of minor thirds. That is, these compromised notes may sometimes conflict against the purer intonation of good soloists, and it is the keyboardist's job to be tastefully retiring.

42 That is, the measurement system itself is oriented toward 20th-century expectations and electronic tuning devices, rather than the standard 18th-century 55-note cycle. Deviations in 'cents' bias us to look for the wrong things, and to judge quality by a historically inappropriate set of musical standards.

43 To appreciate this, spend plenty of time playing and singing in regular ⅙ comma first.

44 Experts in the fields of musical perception and pedagogy will surely have more to say here. At this point I can only report my own impression and expectations: that instrumental melodies should sound reasonably vocal, and any unduly flat notes in leaps (especially on organs) simply sound to me like poorly prepared singing.

45 D. Devie, *Le tempérament musical: philosophie, histoire, théorie et pratique* (Béziers, 1990), p.136. Devie credits the discovery of this connection to Jean Bosquet.

46 Unfortunately for Kellner's invention, the problem is endemic *to that pattern* of Werckmeister III, and it dooms any efforts at moderation. The problem is the ultimately unworkable organization of enharmonic notes: these temperaments fail most noticeably when flat-side music is played in them.

47 I will merely add: esotericism itself fails as 'scientific' inquiry for the same technical reason that division by zero is forbidden from algebra. The improper steps invalidate reliability: they transform any convenient premises, coincidences and truisms into any arbitary and non-unique result the user chooses to force. Occam's Razor is disdained.

48 BuxWV 156 in F, 159 in C minor, and 176 in B♭.

49 Readers are encouraged to *play* extensively in regular ¼, ⅕ and ⅙ comma temperaments with disposition E♭–G♯, to hear and understand the handling of enharmonically wrong sharps and the E♭–G♯ wolf.

50 Buxtehude's harpsichord music stays even more conservatively away from A♭, E♯ and B♯, but has plenty of D♯s and A♯s.

51 Mark Lindley's assertions in that regard have been correct, over many years: each key has a subtly different set of resources.

52 See, for example, C. Ystad *et al.*, 'Influence of rhythmic, melodic and semantic violations in language and music on the electrical activity in the brain' from the Stockholm Music Acoustics Conference 2003, http://www.speech.kth.se/smac03/

53 Cent measurements remain consistent through octave transpositions. This is different from the beat rates that keyboard tuners count; beat rates arise from the different vibrational frequencies of the notes, and double at every octave.

54 Such a balance is not true of the regular mean-tone temperaments, where there are only two sizes of semitones, but the difference in their sound is obvious.

55 Four-note and seven-note systems of *solfège* are still in use today among singers from shaped-note hymnals. Two prominent hymnals that include 'rudiments of music' sections (through *solfège*) are the 25th edition (1993) of Joseph Funk, *The Harmonia Sacra* (1/1832), and Thomas Denson's 1934 edition of *The Sacred Harp*.

56 This is the basis of the keyboard layout itself: with seven natural notes and five raised accidentals.

57 Bach knew the hexachord system from Athanasius Kircher, *Musurgia universalis* (1649), and perhaps from earlier documents or pieces as well (Byrd, Bull *et al.*). This book by Kircher includes Froberger's first published composition, the hexachord fantasy 'Ut Re Mi Fa Sol La'. From the Guidonian Hand, '*ut re mi*' was a system of rudimentary musical instruction. The naming of the notes is from an acrostic of the plainchant 'UT queant laxis / REsonāre fibris / MIra gestorum / FAmuli tuorum, / SOLve polluti / LAbii reatum, / Sancte Ioannes.' *Ut/do, re, mi, fa, sol, la* and *si* are still the standard note-names in romance languages today.

58 These two minor hexachords each have only one note (B♭ or B: respectively 'soft B' or 'hard B') outside the regular mean-tone set (55-division).

59 Modulation in tonal music is akin to 'mutation' in the older system of hexachords: the substitution of half the current set of notes to half of a neighbouring set.

60 As Lindley explained in his Stanford article and several later articles, for all practical purposes the total within each such set (i.e. the diesis) is $^{21}/_{12}$ PC, equivalent in practice to $^{21}/_{11}$ SC or 21 schismas. To see how this works, consider equal temperament in which all these numbers are 7s. If we move any note up or down, we improve one major 3rd while another in its set is made worse, an equal and opposite reaction, as the diesis must be distributed somewhere among the three intervals. Therefore, the total always remains the same, with average 7. See also this same analysis for approximately 40 temperaments as the web supplement for part 1.

61 See the web appendix for part 1.

62 The harshness to which we are accustomed comes from Werckmeister's, Vallotti's, Kellner's and other 'well temperaments' that have especially sharp major 3rds in those dominant chords; but, it does not have to be there.

63 The small C of the temperament diagram itself masquerades as merely a calligraphic capitalization stroke on the bigger C of 'Clavier'. The occurrence of such C-like hook strokes elsewhere (for example, above Bach's opening K in the *Entwurff*, on a C of the title-page of Concerto BWV 1043, and several occurrences on Altnickol's title-page of *WTC* 2) throws sleuths off the steganographic trail that the C belongs *also* with the diagram, which it touches.

64 It is important to study *immediate* thematic elements, audible features.

65 G–D–C seems consonant due to the *regularity* of the 5ths and the C major diatonic context. Then, the playing of B (resolving C) with the G two octaves lower creates a rapid beat rate that draws attention to itself. This is the final resolution into the tonic, highlighted as well by the sudden absence of semiquavers.

66 The fantasia in E♭ by Pachelbel makes similar demands. Bars 14–15 of the B♭ minor prelude are an excellent test spot for A♭–C and D♭–F, exposed. They make an otherwise fine temperament such as Neidhardt's 'Third-circle #4' (see Barbour) sound a bit rough and questionable.

67 'Cents' had not been invented, of course, and are merely a convenient modern way to measure this. But the special *quality* of these small semitones is immediately obvious in listening closely to the temperament.

68 Lest I be accused of esotericism, there is no number-juggling here, and numeric values in cents have no mystical properties. The point is: close listening at a harpsichord reveals which semitones in a temperament are 'small' and which are 'large', and this fugue gives all the small (and subtly different) semitones a thorough workout in Bach's temperament.

69 The chromatic semitones of regular ⅙ comma are 86 cents (in ⅙ PC) or 89 cents (in ⅙ SC): so small as to be grating on the ear. This agrees with Tosi's insistence in his

vocal tutor that singers must never use chromatic semitones within ornaments, but only the diatonic (larger) ones.

70 I have confirmed this suggested function in practice: first setting Bach's temperament by ear (the entire instrument in 13 minutes) and then playing through these two pieces immediately, to test for any problems in the temperament.

71 A handy memory aid to set Bendeler III, from an A fork, is the pattern of vowels in 'PURPLE STREET'.

72 *New Bach reader*, ed. David, Mendel and Wolff no.94; *Bach-Dokumente*, iii, no.801: C. P. E. Bach providing notes to Forkel for the biography. See also Forkel's use of this: *New Bach reader*, p.436 (Forkel's biography, last page of chap.3).